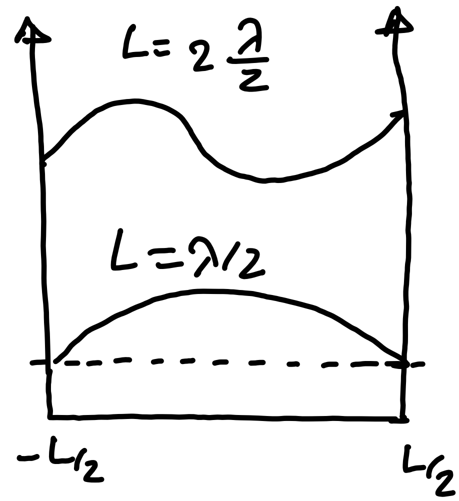
Note Title 1/16/2008

Infinite QW



$$\psi = A e^{ikx} + B e^{-ikx}$$

$$\boxed{L = n\frac{\lambda}{2}} \rightarrow \lambda = \frac{2L}{n}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{2L}{n}} = \frac{\pi n}{L}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow E_n = \frac{\hbar^2}{2m}\,\frac{n^2\pi^2}{L^2}$$

Check B-C. :

$$A e^{-ikL/2} + B e^{ikL/2} = 0$$

$$\rightarrow \quad A e^{-in\pi/2} + B e^{in\pi/2} = 0 \quad \rightarrow$$

$$\begin{cases} n \text{ Even}: & A(1) + B(1) = 0 \Rightarrow A = -B \\ n \text{ odd}: & A(-i) + B(i) = 0 \Rightarrow A = B \end{cases} \Rightarrow$$

$$\begin{cases} n \text{ Even}: & \psi(x) = A\left(e^{-ikx} - e^{ikx}\right) = \underbrace{-2iA}_{C} \sin kx \\ n \text{ odd}: & \psi(x) = A'\left(e^{-ikx} + e^{ikx}\right) = \underbrace{2A'}_{C'} \cos kx \end{cases}$$

Eigenfunct. must be normalized:

$$c^2 \int_{-L/2}^{L/2} \sin^2 kx \, dx = c^2 \int_{-L/2}^{L/2} \sin^2 \frac{n\pi}{L} x \, dx = 1$$

$$\Rightarrow c^2 \left(\frac{L}{2} \right) = 1 \Rightarrow c = \sqrt{\frac{2}{L}}$$

Similarly for $c' = \sqrt{\frac{2}{L}} \Rightarrow$

$$\begin{cases} \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x & n \text{ even (odd parity)} \\ \psi(x) = \sqrt{\frac{2}{L}} \cos \frac{n\pi}{L} x & n \text{ odd (even parity)} \end{cases}$$

We can combine these in a single form:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \left(\frac{n\pi}{L} \left(x + \frac{L}{2} \right) \right) \quad \text{all } n$$

The complete wavefunction is:

$$\psi_n(x,t) = \psi_n(x) \, e^{-i\omega_n t} \qquad E_n = \hbar \omega_n$$

$$\boxed{\psi_n(x,t) = \sqrt{\frac{2}{L}} \sin \left(\frac{n\pi}{L} \left(x + \frac{L}{2} \right) \right) e^{-i\omega_n t}}$$

Current density in QM

From Maxwell's equ. we know that:

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot J \qquad \rho(r,t)\,,\ J(r,t)$$

In QM: $n = |\psi|^2$ Particle density

$$\rho = en = e|\psi|^2$$

$$\frac{\partial \rho}{\partial t} = e\frac{\partial}{\partial t}(\psi^*\psi) = e\underbrace{\frac{\partial \psi^*}{\partial t}}_{\frac{1}{-i\hbar}\left(-\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^*\right)}\psi + e\psi^*\underbrace{\frac{\partial \psi}{\partial t}}_{\frac{1}{i\hbar}\left(-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\right)} \Rightarrow$$

- is for c.c.

$$\frac{\partial \rho}{\partial t} = \frac{e}{i\hbar}\left[\frac{\hbar^2}{2m}\left(\psi\nabla^2\psi^* - \psi^*\nabla^2\psi\right) - V\cancel{\psi^*\psi} + V\cancel{\psi^*\psi}\right]$$

$$= \frac{e\hbar}{i2m}\left(\psi\nabla^2\psi^* - \psi^*\nabla^2\psi\right)$$

$$= \frac{-ie\hbar}{2m}\vec{\nabla}\cdot\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right) = -\nabla\cdot J$$

$$\Rightarrow \boxed{\vec{J} = \frac{ie\hbar}{2m}\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right)}$$